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DETERMINATION OF THE SURFACE STATE OF METAL POWDER PARTICLES BY SCATTERING OF LIGHT

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DETERMINATION OF THE SURFACE STATE OF METAL POWDER PARTICLES BY SCATTERING OF LIGHT **/69

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Quantitative characteristics of the degree of roughness of metal powder particles, assumed to be convex and of equal disperseness, determined from the scattering of light of wavelength shorter than the size of surface irregularities are derived, with formulas for the scattering indicatrices in diffraction, reflection, and absorption. The diffraction part of the scattering is used as dispersity control of the investigated particles. Author

In practical powder metallurgy the profile of the powder particles is often studied experimentally under the optical or electron microscope [see, for instance, Bibl.(1)]. In this paper, we will establish the possibility of introducing objective quantitative characteristics describing the degree of roughness of the powder particles involved, by making use of the varying scattering power of the surface of solids with varying number of surface irregularities of sizes several times as great as the wavelength of the radiation employed. It should be remembered, in this connection, that the powders compared here have particles with similar "rough" shapes, but of different degree of roughness.

Let us assume for simplicity that the particles of the powders under study are convex and substantially of the same disperseness. If visible light is

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used to examine the ordinary powders used in powder metallurgy, then the relation $x \gg 1$ will hold (where $x = \frac{2\pi a}{\lambda}$, a = particle radius, λ = wavelength of the light); in other words, the approximation of geometric optics will obtain. It is well known that light incident on any object undergoes diffraction, reflection (primary or after a series of refractions), and absorption. Let us analyze each of these phenomena and see the results of their study.

1. Diffraction. If the incident light is considered a plane wave and we view the diffraction figure at a distance far greater than the dimensions of the scattering object, we have the case of Fraunhofer diffraction. The intensity of the diffracted light is determined by the formula (Bibl.2):

$$I = \frac{di}{dS} = \frac{G^2}{\lambda^2 r^2} I_0 \, | D(\theta, \varphi) |^2, \tag{1}$$

where di is the quantity of radiant energy incident (as a result of diffraction at the particle) on the elementary area dS perpendicular to the ray; G is the area of the greatest cross section of the scattering particle perpendicular to the incident light; r is the distance from the particle to the elementary area dS; I_0 is the intensity of the incident light; $D(\theta, \varphi)$ is a function describing (with an accuracy to within factors independent of the polar angle θ and the azimuth φ) the amplitude of the diffracted wave. If the shape of the particles is close to spherical, then the amplitude distribution of the scattered light /70 will be close to the law

$$D(\theta, \varphi) = \frac{2J_1\left(\frac{2\pi a}{\lambda} \sin\theta\right)}{\frac{2\pi a}{\lambda} \sin\theta}, \tag{2}$$

where $J_1(z)$ is a Bessel function.

As indicated by eq.(2), in this case the amplitude (and intensity) of the diffracted wave will not depend on the azimuth. We note that the difference between the diffraction figure given here and that given by eq.(1), taking account of eq.(2), can serve as a criterion of the deviation of the particles from spherical shape. For this purpose, for instance, we might use the ratio of the maxima in the second and first diffraction rings. Let us evaluate the minimum distance between object and screen that will permit the first dark ring (i.e., intensity minimum) to appear in the area cut out by the primary beam. The first root of $J_1(z)$ reads

$$z_1 = 0.61 \cdot 2\pi.$$

Since the diffraction pattern is concentrated at very small θ, we will replace $\sin\theta$ by the argument

$$\frac{2\pi a}{\lambda}\theta_1 = 0.61 \cdot 2\pi$$

(θ_1 is the angle at which the appearance of the first dark ring is expected). Hence $\theta_1 = 0.61\,\frac{\lambda}{a}$. Let the linear dimension of the transverse incident beam be of the order of l. Then the required minimum distance R_1 will obey the inequality

$$\theta_1 R_1 \geqslant l,\ 0.61\,\frac{\lambda}{a} R_1 \geqslant l,$$

hence

$$R_1 \geqslant \frac{la}{0.61\lambda}. \qquad (3)$$

If, for instance, $l \simeq 1$ cm, $a \simeq 100\,\mu$, $\lambda \simeq 0.5\,\mu$, then $R_1 \geq 30$ cm.

It is wellknown that diffraction does not depend on the physical nature of the object nor on the physical state of its surface, but is determined exclusively by the geometry of the cross section of the object in the path of the incident light. The fact that the objects being compared consist of particles of the same "rough" shapes can therefore be checked by the distribution of intensity in the diffracted ray.

2. Reflection. For convex particles considerably greater in linear dimensions than the wavelength λ, the law of reflection from spherical particles will hold (Bibl.2). If the powder examined consisted of ideally reflecting spherical particles, then the diagonal elements of the scattering matrix would be nonzero and would be expressed by the formulas (Bibl.2)

$$-S_1(\theta) = S_2(\theta) = \frac{1}{2} i \frac{2\pi a}{\lambda} \exp\left(2i\sin\tau \cdot \frac{2\pi a}{\lambda}\right), \tag{4}$$

where τ is the angle between the straight line running from the point at which the incident ray meets the surface of the sphere facing the ray, and the tan-/71 gent plane to the sphere at that point, while $\theta = 2\tau$ is the angle of scattering (Fig.1).

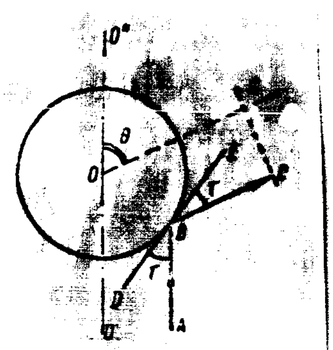

Fig.1 Geometry of Reflection of Light from the Surface of a Spherical Particle
O',O" - direction of incident light; AB||O'O" - same, of incident rays; DE - trace of plane tangent to the sphere at the point of incidence of the ray B; BC - reflected ray; OG - line indicating direction of reflected ray and passing through the center of the particle (BC||OG). The angles are marked on the sketch.

Equation (4) is valid for spherical particles of a substance with Fresnel reflection coefficients equal in absolute value to unity. For the case of a metal sphere, the components of the scattering matrix are obtained on multiplying $S_1(\theta)$ and $S_2(\theta)$ of eq.(4) by the corresponding reflection coefficients: r_1 for polarization perpendicular to the plane of scattering and r_2 parallel to the plane of scattering. Hence, we obtain the expression for the intensity of scattering of natural light:

$$I = \frac{1}{8}\frac{a^2}{r^2}\left(|r_1|^2 + |r_2|^2\right) I_0. \tag{5}$$

To obtain the angular intensity distribution [eq.(5)], we must find the angular dependence of $|r_1|^2$ and $|r_2|^2$. It is well known that, over a wide range of wavelengths of electromagnetic radiation (including the luminous wavelengths), metals have a complex refractive index

$$m = n' - in'', \tag{6}$$

where both n' and n'' are of the order of unity in the visible spectrum region. To calculate the required squares of the moduli of the reflection coefficients, we must use generalized Fresnel formulas for media with complex reflection coefficients [cf., for instance, (Bibl.3)]. Remembering that, in the range of interest here, the magnetic permeability is unity, these formulas will have the form

$$r_1 = \frac{\sqrt{1-\cos^2\tau} - \sqrt{m^2-\cos^2\tau}}{\sqrt{1-\cos^2\tau} + \sqrt{m^2-\cos^2\tau}} \tag{7}$$

$$r_2 = \frac{\sqrt{m^2-\cos^2\tau} - m^2\sqrt{1-\cos^2\tau}}{\sqrt{m^2-\cos^2\tau} + m^2\sqrt{1-\cos^2\tau}} \tag{8}$$

(where $\cos^2\tau$ is used everywhere as the angular quantity, to facilitate the forthcoming transition from τ to the scattering angle θ).

Substituting eq.(6) into eqs.(7) and (8) leads to the following expressions:

$$r_1 = \frac{\sqrt{1-\cos^2\tau} - \sqrt{n'^2 - n''^2 - \cos^2\tau - 2n'n''i}}{\sqrt{1-\cos^2\tau} - \sqrt{n'^2 - n''^2 - \cos^2\tau - 2n'n''i}}; \tag{9}$$

$$r_2 = \frac{\sqrt{n'^2 - n''^2 - \cos^2\tau - 2n'n''i} - \sqrt{1-\cos^2\tau}\,(n'^2 - n''^2 - 2n'n''i)}{\sqrt{n'^2 - n''^2 - \cos^2\tau - 2n'n''i} + \sqrt{1-\cos^2\tau}\,(n'^2 - n''^2 - 2n'n''i)}. \tag{10}$$

The square of the modulus $|r_1|^2 = r_1 r_1^*$ is obtained on multiplying eq.(9) by its complex conjugate /72

$$r_1^* = \frac{\sqrt{1-\cos^2\tau} - \sqrt{n'^2 - n''^2 - \cos^2\tau + 2n'n''i}}{\sqrt{1-\cos^2\tau} + \sqrt{n'^2 - n''^2 - \cos^2\tau + 2n'n''i}}. \tag{11}$$

An analogous operation must be performed on r_2. Unwieldy expressions, omitted here, lead to the following relations:

$$|r_1|^2 = \frac{A_1}{B_1}, \tag{12}$$

where

$$\left.\begin{matrix} A_1 \\ B_1 \end{matrix}\right\} = \frac{1-\cos\theta}{2} + \sqrt{\left[n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)\right]^2 + 4n'^2n''^2} \mp$$
$$\mp \sqrt{1-\cos\theta}\cdot\sqrt[4]{\left[n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)\right]^2 + 4n'^2n''^2} \times \tag{13}$$
$$\times \sqrt{1 + \frac{n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)}{\sqrt{\left[n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)\right]^2 + 4n'^2n''^2}}} = \begin{cases} A_1 \\ B_1 \end{cases} \tag{14}$$

$$|r_2|^2 = \frac{A_2}{B_2}, \tag{15}$$

where

$$\left.\begin{matrix} A_2 \\ B_2 \end{matrix}\right\} = \sqrt{\left[n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)\right]^2 + 4n'^2n''^2} + \frac{1-\cos\theta}{2}\left[(n'^2 - n''^2)^2 + \right.$$
$$\left. + 4n'^2n''^2\right] \mp \sqrt{1-\cos\theta}\cdot\sqrt[4]{\left[n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)\right]^2 + 4n'^2n''^2} \times$$
$$\times \left\{ (n'^2 - n''^2)\sqrt{1 + \frac{n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)}{\sqrt{\left[n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)\right]^2 + 4n'^2n''^2}}} + \right.$$
$$\left. + 2n'n''\sqrt{1 - \frac{n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)}{\sqrt{\left[n'^2 - n''^2 - \frac{1}{2}(1+\cos\theta)\right]^2 + 4n'^2n''^2}}} \right\} = \tag{16}$$
$$= \begin{cases} A_2 \\ B_2 \end{cases}. \tag{17}$$

[in eqs.(13), (14), (16), (17) the minus sign relates to A_1, A_2, and the plus sign to B_1, B_2 respectively].

The scattering indicatrix may be characterized by the angular dependence of the quantity:

$$H = \frac{4r^2}{a^2}\frac{I}{I_0} = \frac{1}{2}(|r_1|^2 + |r_2|^2). \tag{18}$$

The above relations must be considered more or less true for smooth reflecting particles. However, if the surface of these particles is covered by a considerable number of irregularities of a size comparable with the wavelength of the radiation used, then the scattering of the incident light by the particles will be diffuse, and allowance must be made for this by an appropriate factor. For example, if the diffuse reflection obeys the Lambert law, then the factor /73

$$f_L(\theta) = \frac{1}{\pi}(\sin\theta - \theta\cos\theta) \tag{19}$$

must be used. If, instead, it is determined by the Seelinger law, then the factor to be applied will be (Bibl.4)

$$f_S(\theta) = 1 - \cos\frac{\theta}{2}\cot\frac{\theta}{2}\ln\tan\frac{\pi-\theta}{4}. \tag{20}$$

It should be remembered that eqs.(19) and (20) give so-called "unnormed" laws of diffuse reflection and therefore permit an investigation of the form of the indicatrix but by no means of the absolute values of $H(\theta)\cdot f(\theta)$. This should satisfy us, however, since the criteria proposed will be the ratios of the quantities $H(\theta)$,

$$H_L(\theta) = H(\theta)\cdot f_L(\theta) \tag{21}$$

and

$$H_S(\theta) = H(\theta)\cdot f_S(\theta) \tag{22}$$

at any two angles θ, without depending on the constant factors by which $f_L(\theta)$ and $f_s(\theta)$ may have been multiplied. Numerical tables corresponding to each of the two laws (19) and (20) are given elsewhere (Bibl.4).

As an example, we calculated three scattering indicatrices for iron powder with fairly coarse particles ($x \gg 1$), at monochromatic unpolarized light of wavelength $\lambda = 0.668\ \mu$. The refractive index in this case (Bibl.2) was $m = 1.70 - 1.84\ i$.

Figure 2 shows all three indicatrices (without the diffractional part). No allowance has been made for the end effects (Bibl.2). The diagram indicates that iron particles with a smooth surface scatter almost isotropically in the back hemisphere ($\theta \geq 90^\circ$), while scattering in the front hemisphere ($\theta < 90^\circ$) increases substantially, reaching a maximum at $\theta = 90^\circ$. The diagram also shows (at half-scale) the scattering indicatrices of the same iron powder particles with a surface that diffusely reflects light according to Lambert and according to Seelinger. The two indicatrices resemble each other. The most substantial differences are noted in the range of angles 40 to 80°.

Having determined the scattering indicatrices, we can then select criteria for characterizing the surface state of the particles under examination. For example, such a criterion might be

$$\frac{H(60^\circ)}{H(170^\circ)}$$

For a theoretical scattering indicatrix, this quantity takes the following numerical values: /74

Law of Reflection	Mirror Reflection	According to Lambert	According to Seelinger
$\frac{H(60^\circ)}{H(170^\circ)}$	1.08	0.12	0.196

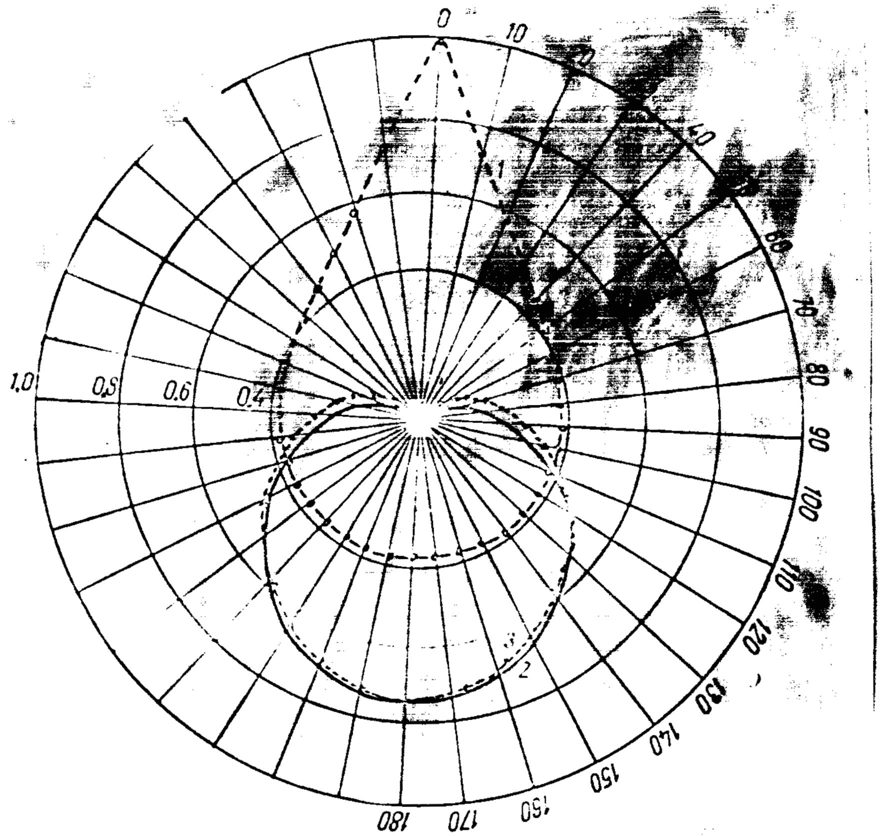

Fig.2 Scattering Indicatrices of Monochromatic (λ = 0.668 μ) Unpolarized Light on Iron Powder
1 - $H(\theta)$ = mirror reflection; 2 - $2H_L(\theta)$ = diffuse reflection according to Lambert; 3 - $2H_s(\theta)$ = = diffuse reflection according to Seelinger

3. Absorption. For $x \gg 1$, the refracted ray is completely absorbed in the metal powder and does not participate in the formation of the diffraction pattern (Bibl.2). Thus, diffraction and reflection should be taken into account, and this has been done above.

BIBLIOGRAPHY

1. Fedorchenko, I.M. and Andriyevskiy, R.A.: Principles of Powder Metallurgy (Osnovy poroshkovoy metallurgii). Izd. Akad. Nauk UkrSSR, Kiev, 1963.

2. van de Hulst, G.: Scattering of Light by Small Particles (Rasseyaniye sveta

malymi chastitsami). Izd. Inostr. Lit., Moscow, 1961.

3. Sokolov, A.V.: Optical Properties of Metals (Opticheskiye svoystva metallov). Fizmatgiz, Moscow, 1961.

4. Schoenberg, E.: Handb. Astrophys., Vol.2, No.1, p.255, 1929.